Experiments, decision rules, and costly information acquisition

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Motivation

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$$\underset{(\text{decision rule}, \text{experiment})}{\text{maximize}} \quad [\text{benefit} - \text{cost}]$$

- Decision rule: signal-dependent actions
- Experiment: state-dependent signal distributions

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- Benefit: ex ante expected payoff w/ information
- Cost: money/time/fatigue to generate/process information

Objective

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- Axiomatic model of costly information acquisition
    - Bayesian DM + information cost
- Primitive: $\succsim$ over $(\text{decision rule}, \text{experiment})$
- Characterize several models that differ in cost structures:
    - **Today**: 1. general 2. posterior separable

1 **Model** 2 Characterization 3 Summary

Primitives

- $\mathscr{E}$: set of ***experiments*** $e\colon \Omega \longrightarrow \Delta(S),\ \omega \mapsto e_\omega$
  - Ω: finite set of ***states***
  - S: Polish space of ***signals***
  - $\operatorname{supp} e \equiv \bigcup_{\omega \in \Omega} \operatorname{supp} e_\omega$

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  - $\mathscr{H}$: set of (***AA***) ***acts*** $h : \Omega \to \Delta(X)$, X: set of outcomes
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- $\succsim$: preference over ***strategies*** $(\delta, e) \in \mathscr{D}\times\mathscr{E}$

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4. A state is resolved & DM receives a payoff

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$$r^u_\omega(\delta, e) = \int u(\delta^e_s(\omega))\,\mathrm{d}e_\omega(s) = u\Big(\int \delta^e_s(\omega)\,\mathrm{d}e_\omega(s)\Big) = u((\delta * e)_\omega)$$

where

$$\delta * e \equiv \Big(\int \delta^e_s(\omega)\,\mathrm{d}e_\omega(s)\Big)_{\omega\in\Omega}$$

(the ***induced act by*** (δ, e))

Definition

An (***information***) ***cost function*** is any continuous $c\colon \mathcal{E} \to \mathbb{R}_+$ s.t.

1. $e \succcurlyeq_{\mathrm{B}} f \implies c(e) \geq c(f)$;
2. $c(e^0) = 0$ for each uninformative $e^0 \in \mathcal{E}$.

- Topology over experiments is induced by distr. over posteriors
  - Topology is prior-independent
- $\succcurlyeq_{\mathrm{B}}$: ***Blackwell order*** on $\mathcal{E}$
- e^0 is ***uninformative*** $\overset{\text{def}}{\iff} e^0_\omega = e^0_{\omega'},\ \ \forall(\omega, \omega') \in \Omega^2$

Aggregators

Definition

An ***aggregator*** is any continuous $W : T \times K \longrightarrow \mathbb{R}$ s.t.

- T and K are real intervals;
- $W(\cdot, k)$ is increasing for each $k \in K$;
- $W(t, \cdot)$ is decreasing for each $t \in T$.

- Additive aggregator: $(t, k) \mapsto t - k$
- Multiplicative aggregator: $(t, k) \mapsto \mathrm{e}^{-k} t$

Definition

A ***costly information acquisition representation of*** $\succsim$ is $\langle u, \mu, c, W \rangle$ s.t.

- u is a vNM function;
- μ is a full support prior;
- c is a cost function;
- W is an aggregator;
- $\succsim$ is represented by

$$V(\delta, e) = W\left(\int r_{\omega}^{u}(\delta, e)\, \mathrm{d}\mu(\omega), c(e)\right).$$

Posterior separability

Definition

A ***posterior separable representation of*** $\succsim$ is $\langle u, \mu, H \rangle$ s.t.

- u is a vNM function;
- μ is a full support prior;
- H is a convex function on $\Delta(\Omega)$;
- $\succsim$ is represented by

$$V(\delta, e) = \int r^u_\omega(\delta, e)\,\mathrm{d}\mu(\omega) - c(e),$$
$$c(e) = \int H(\mu^e_s)\,\mathrm{d}e_\mu(s) - H(\mu).$$

1 Model

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Axioms

A1—Regularity

$\succsim$ is nondegenerate, complete, transitive, and continuous.

- Topology over strategies:

Two strategies are "close" $\iff$

the induced acts and experiments are "close"

A2—Statewise dominance

For each $((\delta,\gamma),e) \in \mathscr{D}^2 \times \mathscr{E}$,

1. $\delta \geq^e_{\mathrm{D}} \gamma \implies (\delta,e) \succsim (\gamma,e)$;
2. $\delta >^e_{\mathrm{D}} \gamma \implies (\delta,e) \succ (\gamma,e)$.

- $\delta \geq^e_{\mathrm{D}} \gamma \iff ((\delta * e)_\omega, e) \succsim ((\gamma * e)_\omega, e) \quad \forall \omega \in \Omega$
- $\delta >^e_{\mathrm{D}} \gamma \iff \delta \geq^e_{\mathrm{D}} \gamma \;\&\; ((\delta * e)_\omega, e) \succ ((\gamma * e)_\omega, e) \quad \exists \omega \in \Omega$

A3—Information monotonicity

For each $((\delta, \gamma), (e, f)) \in \mathscr{D}^2 \times \mathscr{E}^2$, if

$$\delta * e = \gamma * f \quad \text{and} \quad f \succcurlyeq_{\mathrm{B}} e,$$

then $(\delta, e) \gtrsim (\gamma, f)$.

- More informative $\implies$ more costly

A4—Cost consistency

For each $((\delta, \tilde{\delta}, \gamma, \tilde{\gamma}), (e, f)) \in \mathscr{D}^4 \times \mathscr{E}^2$, if

$$\delta * e = \gamma * f \quad \text{and} \quad \tilde{\delta} * e = \tilde{\gamma} * f \quad \text{and} \quad (\delta, e) \succsim (\gamma, f),$$

then $(\tilde{\delta}, e) \succsim (\tilde{\gamma}, f)$.

- Cost does not depend on decision rules

Matrix notation of experiments

Let $\Omega = \{1, \dots, m\}$

If $e \in \mathscr{E}$ satisfies $\operatorname{supp} e = \{s_1, \dots, s_n\}$, then

$$e \simeq \begin{bmatrix} e_1(s_1) & \cdots & e_1(s_n) \\ \vdots & \ddots & \vdots \\ e_m(s_1) & \cdots & e_m(s_n) \end{bmatrix}, \qquad \sum_{i=1}^{n} e_\omega(s_i) = 1, \quad e_\omega(s_i) \geq 0$$

- e is identified w/ a row stochastic matrix

Concatenation of experiments

- If $\operatorname{supp} e \cap \operatorname{supp} f = \emptyset$,

$$\lambda\text{-mixture of } e \text{ and } f = \text{“}\lambda\text{-concatenation of } e \text{ and } f\text{”}$$

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$$\lambda\text{-mixture of } e \text{ and } f = \text{“}\lambda\text{-concatenation of } e \text{ and } f\text{”}$$

- In matrix notation, if $\operatorname{supp} e = \{s_1, \ldots, s_n\}$ and $\operatorname{supp} f = \{\tilde{s}_1, \ldots, \tilde{s}_k\}$,

$$\lambda e + (1-\lambda) f \simeq \left[\lambda e \;\middle|\; (1-\lambda) f\right]$$

$$= \left[\begin{array}{ccc|ccc} \lambda e_1(s_1) & \cdots & \lambda e_1(s_n) & (1-\lambda) f_1(\tilde{s}_1) & \cdots & (1-\lambda) f_1(\tilde{s}_k) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \lambda e_m(s_1) & \cdots & \lambda e_n(s_n) & (1-\lambda) f_m(\tilde{s}_1) & \cdots & (1-\lambda) f_m(\tilde{s}_k) \end{array}\right]$$

A5—Equivalent concatenation independence

For each $(\lambda, \delta, (e, e', f)) \in (0, 1] \times \mathscr{D}_{\text{inv}} \times \mathscr{E}^3$ w/

$e \sim_{\text{B}} e'$ and $(\operatorname{supp} e \cup \operatorname{supp} e') \cap \operatorname{supp} f = \emptyset$,

$$(\delta, e) \succsim (\delta, e') \iff (\delta, \lambda e + (1-\lambda) f) \succsim (\delta, \lambda e' + (1-\lambda) f).$$

- $\mathscr{D}_{\text{inv}}$: set of ***invariant*** decision rules
  - $\delta_s^e = \delta_s^f$ for each $((e, f), s) \in \mathscr{E}^2 \times S$
- $e \sim_{\text{B}} e' \implies \lambda e + (1-\lambda) f \sim_{\text{B}} \lambda e' + (1-\lambda) f$ under support disjointness
- Mixture affects only on induced acts

Theorem 1

$\succsim$ *satisfies A1–A5* $\iff$

$\succsim$ *has a costly information acquisition representation.*

- A1—*Regularity*
- A2—*Statewise dominance*
- A3—*Information monotonicity*
- A4—*Cost consistency*
- A5—*Equivalent concatenation independence*

Posterior separable representation

Theorem 3

$\succsim$ satisfies A1–A3 & A6 $\iff$ $\succsim$ has a posterior separable representation.

A6—Concatenation independence

For each $(\lambda, \delta, (e, e', f)) \in (0,1] \times \mathscr{D}_{\text{inv}} \times \mathscr{E}^3$ w/

~~$e \sim_B e'$ and~~ $(\operatorname{supp} e \cup \operatorname{supp} e') \cap \operatorname{supp} f = \emptyset$,

$$(\delta, e) \succsim (\delta, e') \iff (\delta, \lambda e + (1-\lambda) f) \succsim (\delta, \lambda e' + (1-\lambda) f).$$

- *A6* says signal-wise separability + separability of benefit and cost
- *Cost consistency* is implied by the other axioms

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SUMMARY

This paper characterizes

- Bayesian DM + costly information acquisition
- Bayesian DM + a posterior separable cost

Literature

- **Preference over menus**:

  menu choice $\longrightarrow$ signal arrival $\longrightarrow$ choice from menu

  - de Oliveira (2014), de Oliveira · Denti · Mihm · Ozbek (2018), Higashi · Hyogo · Takeoka (2025)

- **Stochastic choice**:

  signal arrival $\longrightarrow$ choice from a menu $\longrightarrow$ stochastic choice

  - Caplin · Dean (2015), Chambers · Liu · Rehbeck (2020), Denti (2022)

- **Statistical decision**:

  - Furtado (2024)—same domain · no cost of information