Randomization and ambiguity perception

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Motivation

- People dislike unknown probabilities—***ambiguity aversion***
  - $\langle$unambiguous event$\rangle \succ \langle$ambiguous event$\rangle$
  - Incompatible w/ SEU ©Ellsberg (1961)
  - Motivated various ambiguity models MEU, CEU, smooth ambiguity, ...

This paper

$$\langle \text{less \# of ambiguous events} \rangle \succ \langle \text{more \# of ambiguous events} \rangle$$

- e.g., Machina's (2009) paradox Lab experiment: L'Haridon · Placido (2010)
  - Incompatible w/ many ambiguity models MEU, CEU, smooth ambiguity, ...

Main axiom

$$\langle \text{less \# of ambiguous events} \rangle \succ \langle \text{more \# of ambiguous events} \rangle$$

Ex ante aversion to randomization ***informal**

DM dislikes randomization ***over*** acts.

- **Idea**: randomization increases # of relevant ambiguous events
- Domain = { lotteries over AA acts } ⊃ { AA acts }
  - AA act : state ↦ outcome distribution

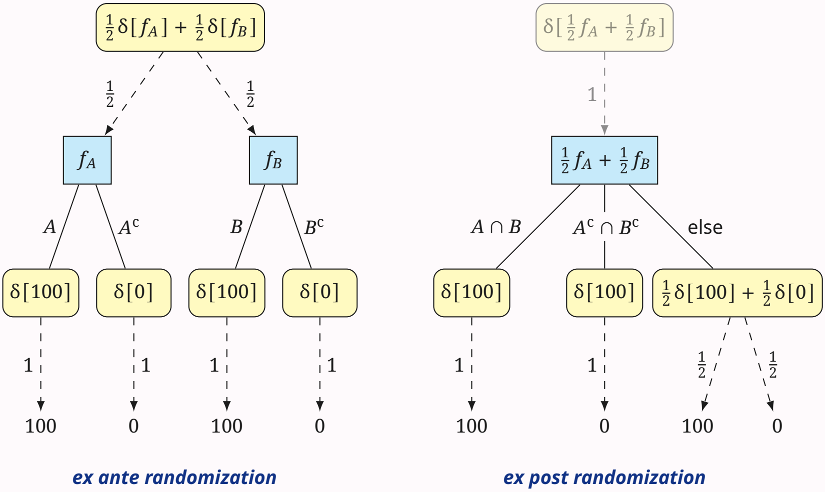
Example

Compare the two acts & their randomization:

$$f_A = \begin{cases} 100 & \text{if} \geq t^\circ\text{C @Athens} \\ 0 & \text{else} \end{cases} \qquad f_B = \begin{cases} 100 & \text{if} \geq t^\circ\text{C @Beijing} \\ 0 & \text{else} \end{cases}$$

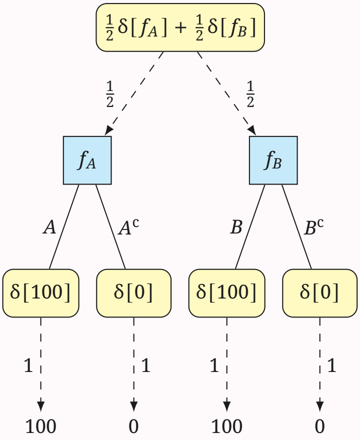
$$\frac{1}{2}\delta[f_A] + \frac{1}{2}\delta[f_B] \neq \frac{1}{2}f_A + \frac{1}{2}f_B$$

Randomization & ambiguous events



$A = [\geq t°\text{C @Athens}]$, $B = [\geq t°\text{C @Beijing}]$

Randomization & ambiguous events



- Both A and B are relevant in the lottery
- $\delta[f_A] \succsim \delta[f_B] \implies \delta[f_A] \succsim \frac{1}{2}\delta[f_A] + \frac{1}{2}\delta[f_B]$

DM dislikes ***ex ante*** *randomization*

Overview

- Explore the implications of *ex ante aversion to randomization*
  - Novel tesable manifestation of

    $\langle$less # of ambiguous events$\rangle \succ \langle$more # of ambiguous events$\rangle$

- Characterize the ***costly ambiguity perception*** model
  - Generalizes MEU—$U_{\text{MEU}}(f) = \min_{\mu \in M} \int u \circ f \, d\mu$ $\quad M$: ambiguity perception
  - DM optimizes ambiguity perception at a cost

- Fully identify the parameters of the model
  - Ex ante randomization is important!

PRIMITIVES

- Ω: finite set of ***states***
- X: set of ***outcomes***
- $\mathscr{F}$: set of ***acts*** $f: \Omega \to \Delta_s(X)$
  - $\Delta_s(X)$: set of finitely supported distributions on X
  - Identify each $p \in \Delta_s(X)$ w/ the constant act $\omega \mapsto p$
- $\succsim$: DM's preference on $\Delta_s(\mathscr{F})$

Definition

A ***costly ambiguity perception representation of*** $\succsim$ is $\langle u, (\mathbb{M}, c)\rangle$ s.t.

- $u\colon \Delta_s(X) \to \mathbb{R}$ is surjective & mixture linear;
- $\mathbb{M}$ is a compact set of nonempty closed convex subsets of $\Delta(\Omega)$;
- $c\colon \mathbb{M} \to \mathbb{R}_+$ is 1. lower semicontinuous; 2. $\min c(\mathbb{M}) = 0$; 3. $M \subseteq M' \implies c(M) \geq c(M')$;
- $\succsim$ is represented by

$$U_{\mathrm{CAP}}(P) = \max_{M \in \mathbb{M}} \left[\int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f) - c(M) \right].$$

Special cases—w/o ex ante randomization

- $\mathbb{M} = \{M\} \rightsquigarrow$ maxmin expected utility ©Gilboa · Schmeidler (1989)

$$U_{\mathsf{MEU}}(\delta[f]) = \max_{M \in \mathbb{M}} \left[\int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f) - c(M) \right]$$

- $c \equiv 0 \rightsquigarrow$ dual-self expected utility ©Chandrasekher et al. (2022)

$$U_{\mathsf{DSEU}}(\delta[f]) = \max_{M \in \mathbb{M}} \left[\int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f) - c(M) \right]$$

- $\mathbb{M} = \{ \mathrm{core}(v) \mid v \in V \} \rightsquigarrow$ optimal ambiguity attitude

V: a set of convex capacities = supermodular set fn.s ©Payró et al. (2026)

$$U_{\mathsf{OAA}}(\delta[f]) = \max_{M \in \mathbb{M}} \left[\int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f) - c(M) \right]$$

Basic axioms

A1—Regularity

$\succsim$ is nondegenerate, complete, transitive, & mixture continuous.

A2—Monotonicity

"$\succsim$ respects statewise dominance."

A7—Unboundedness

"Utility has unbounded range."

Timing axioms

A3—Attraction to ex post randomization

"⟨xp randomization⟩ $\succsim$ ⟨xa randomization⟩."

- Ex post randomization may smooth payoff variability

$$\frac{1}{2}\begin{bmatrix}100\\0\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0\\100\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\delta[100]+\frac{1}{2}\delta[0]\\\frac{1}{2}\delta[100]+\frac{1}{2}\delta[0]\end{bmatrix}$$

A4—Indifference to randomization timing of constant acts

"⟨xp randomization of f & p⟩ $\sim$ ⟨xa randomization of f & p⟩."

- Ex post mixing of p preserves the pattern of payoff variability in f

$$\frac{1}{2}\delta\left[\begin{bmatrix}100\\0\end{bmatrix}\right]+\frac{1}{2}\delta[0]\sim\begin{bmatrix}\frac{1}{2}\delta[100]+\frac{1}{2}\delta[0]\\\delta[0]\end{bmatrix}$$

Ex ante randomization axioms

A5—Ex ante aversion to randomization

For each $(\lambda, (P, Q)) \in [0, 1] \times \Delta_s(\mathscr{F})^2$,

$$P \succsim Q \implies P \succsim \lambda P + (1 - \lambda)Q.$$

- Ex ante randomization increases # of relevant ambiguous events

A6—Independence of constant acts

For each $(\lambda, (P, Q), (p, q)) \in [0,1] \times \Delta_s(\mathscr{F})^2 \times \Delta_s(X)^2$,

$$\lambda P + (1-\lambda)\delta[\boldsymbol{p}] \succsim \lambda Q + (1-\lambda)\delta[\boldsymbol{p}]$$
$$\implies \lambda P + (1-\lambda)\delta[\boldsymbol{q}] \succsim \lambda Q + (1-\lambda)\delta[\boldsymbol{q}].$$

- **Idea**: constant act terms do not affect ambiguity

CHARACTERIZATION

Representation theorem

$\succsim$ *satisfies* A1–A7 $\Longleftrightarrow$ $\succsim$ *has a* CAP *representation.*

- A1—*Regularity*
- A2—*Monotonicity*
- A3—*Attraction to ex post randomization*
- A4—*Indifference to randomization timing of constant acts*
- A5—*Ex ante aversion to randomization*
- A6—*Independence of constant acts*
- A7—*Unboundedness*

Characterization

Corollary

$\succsim$ *satisfies* A1–A4 + *independence* $\iff$ $\succsim$ *has an* MEU *representation.*

- A1—*Regularity*
- A2—*Monotonicity*
- A3—*Attraction to ex post randomization*
- A4—*Indifference to randomization timing of constant acts*
- A5—*Ex ante aversion to randomization*
- A6—*Independence of constant acts*
- A7—*Unboundedness*

UNIQUENESS

Definition

A CAP representation $\langle u, (\mathbb{M}, c)\rangle$ is ***convex*** if $\mathbb{M}$ and c are convex.

Identification theorem

Every CAP preference $\succsim$ has convex rep.s $\langle u, (\mathbb{M}^*, c^*)\rangle$ & $\langle u, (\mathbb{M}^{**}, c^{**})\rangle$ s.t. for each convex rep. $\langle u, (\mathbb{M}, c)\rangle$ of $\succsim$,

1. $\mathbb{M}^* \subseteq \mathbb{M} \subseteq \mathbb{M}^{**}$;
2. $c = c^{**}|_{\mathbb{M}}$.

1. ∃ smallest & largest feasible sets
2. Cost is essentially unique

Smallest/largest representation

- $(u, \mathbb{M}^*)$ represents the ***expected utility core***: smallest feasible set

$$P \succsim^* Q \iff$$

$$\int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f) \geq \int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}Q(f) \quad \forall M \in \mathbb{M}^*$$

$\succsim^*$: the largest subrelation of $\succsim$ satisfying *independence*

cf. Cerreia-Vioglio et al. (2015), Ke · Zhang (2020), . . .

- $(\mathbb{M}^{**}, c^{**})$ is explicitly written as: largest cost structure

$$\mathbb{M}^{**} = \langle \text{the } \supseteq\text{-upward closure of } \mathbb{M}^* \rangle = \operatorname{dom} c^{\star}_{\succsim,u}$$

$$c^{**}(M) = c^{\star}_{\succsim,u}(M) \equiv \sup \left\{ \int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f) - u(\bar{P}) \;\middle|\; P \sim \delta[\bar{P}] \right\}$$

cf. Maccheroni et al. (2006), de Oliveira et al. (2017)

Example

Let $\Omega = \{0, 1\}$ and $X = \mathbb{R}$. Identify $\mu \in [0, 1]$ w/ the prob. of $\omega = 1$.

Consider $\succsim_1$ and $\succsim_2$ represented by $\langle u, (\mathbb{M}_i, c_i) \rangle$:

$$u(\delta[x]) = x, \quad \mathbb{M}_1 = \{\{\tfrac{1}{2}\}\}, \quad \mathbb{M}_2 = \{[0, \tfrac{1}{2}], [\tfrac{1}{2}, 1]\}, \quad c_1 = c_2 \equiv 0.$$

While $\succsim_1$ is neutral, $\succsim_2$ is averse to ex ante randomization.

Nevertheless, $\succsim_1$ and $\succsim_2$ coincide on the set of acts:

$$U_2(\delta[h]) = \left[\tfrac{1}{2}h(0) + \tfrac{1}{2}(h(0) \wedge h(1))\right] \vee \left[\tfrac{1}{2}(h(0) \wedge h(1)) + \tfrac{1}{2}h(1)\right]$$
$$= \tfrac{1}{2}(h(0) \wedge h(1)) + \tfrac{1}{2}(h(0) \vee h(1)) = U_1(\delta[h]) \quad \forall h \in \mathscr{F}. \quad \Diamond$$

Comparative ambiguity aversion

Definition **cf. Ghirardato · Marinacci (2002)**

$\succsim_1$ is ***more ambiguity-averse than*** $\succsim_2$ if

$$\delta[p] \succ_2 P \implies \delta[p] \succ_1 P \qquad \forall (P, p) \in \Delta_s(\mathscr{F}) \times \Delta_s(X).$$

Proposition

The following are equivalent.

a. $\succsim_1$ *is more ambiguity-averse than* $\succsim_2$.

b. $u_1 \approx u_2, \ \ \mathbb{M}_1^{**} \subseteq \mathbb{M}_2^{**}, \ \ \& \ \ c_1^{**} \geq c_2^{**}|_{\mathbb{M}_1^{**}}$.

- Cost function describes ambiguity attitudes

Comparative aversion to additional ambiguity

Definition

$\succsim_1$ is ***more averse to additional ambiguity than*** $\succsim_2$ if

$$\lambda\delta[p] + (1-\lambda)Q \succ_2 \lambda P + (1-\lambda)Q$$
$$\implies \lambda\delta[p] + (1-\lambda)Q \succ_1 \lambda P + (1-\lambda)Q$$

for each $(\lambda, (P, Q), p) \in [0,1] \times \Delta_s(\mathscr{F})^2 \times \Delta_s(X)$.

- $\implies$ ($\not\Leftarrow$) $\succsim_1$ is more ambiguity-averse than $\succsim_2$

Comparative aversion to additional ambiguity

- Define $\mathscr{C}_i : \Delta_s(\mathscr{F}) \rightrightarrows \mathbb{M}_i$ by

$$\mathscr{C}_i(P) = \arg\max_{M \in \mathbb{M}_i} \left[\int \left(\min_{\mu \in M} \int u_i \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f) - c_i(M) \right]$$

Proposition

The following are equivalent.

a. $\succsim_1$ *is more averse to additional ambiguity than* $\succsim_2$.

b. $u_1 \approx u_2$ &

$$\forall P \in \Delta_s(\mathscr{F}), \; M_1 \in \mathscr{C}_1(P) \implies \exists M_2 \in \mathscr{C}_2(P) : M_2 \subseteq M_1.$$

b. "DM 2 always reduces ambiguity perception more than DM 1"

Summary

- Dm who dislikes increases in ambiguity is studied
- Dm is averse to randomization over acts

$$P \succsim Q \implies P \succsim \lambda P + (1-\lambda)Q$$

- Dm behaves as if optimizing her ambiguity perception at a cost

$$U_{\text{CAP}}(P) = \max_{M \in \mathbb{M}} \left[\int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f) - c(M) \right]$$

- Parameters are uniquely identified & given behavioral meanings
- In the paper . . . ,
  - characterization of several special cases — DSEU, OAA, . . .
  - more on comparatives — absolute notion, XA rand., . . .
  - Machina's (2009) paradoxes: reflection & 50–51 examples

Choice of ambiguity perception w/o ex ante randomization

- Chandrasekher · Frick · Iijima · Le Yaouanq (2022) ⟵ NO COST
- Payró · Takeoka · Xia (2026) ⟵ CHOICE OF CAPACITIES

Ambiguity aversion & ex ante randomization

- Saito (2015), Ke · Zhang (2020)
- Their DM believes even ex ante randomization reduces ambiguity

$$U_{\mathrm{DMEU}}(P) = \min_{M \in \mathbb{M}} \int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f)$$

Models consistent w/ Machina's examples

- Siniscalchi (2009), Dillenberger · Segal (2015), He (2021), . . .

Summary

- DM who dislikes increases in ambiguity is studied
- DM is averse to randomization over acts

$$P \succsim Q \implies P \succsim \lambda P + (1-\lambda) Q$$

- DM behaves as if optimizing her ambiguity perception at a cost

$$U_{\mathsf{CAP}}(P) = \max_{M \in \mathbb{M}} \left[\int \left(\min_{\mu \in M} \int u \circ f \, \mathrm{d}\mu \right) \mathrm{d}P(f) - c(M) \right]$$

- Parameters are uniquely identified & given behavioral meaning

Appendix

A Special cases

B More on comparatives

C Reflection example

D 50–51 example

A8—Strong independence of constant acts

For each $(\lambda, (P, Q), p) \in [0,1] \times \Delta_s(\mathscr{F})^2 \times \Delta_s(X)$,

$$P \succsim Q \iff \lambda P + (1-\lambda)\delta[p] \succsim \lambda Q + (1-\lambda)\delta[p].$$

Corollary

$\succsim$ satisfies A1–A7 + A8 $\iff$ $\succsim$ has a DSEU representation.

Optimal ambiguity attitude

A9—Indifference to randomization timing of comonotonic acts

For each $((\kappa,\lambda), P, (f,g)) \in [0,1]^2 \times \Delta_s(\mathscr{F}) \times \mathscr{F}^2$,

if $\delta[f(\omega)] \succsim \delta[f(\omega')]$ is equivalent to $\delta[g(\omega)] \succsim \delta[g(\omega')]$, then

$$\kappa\delta[\lambda f + (1-\lambda)g] + (1-\kappa)P \sim \kappa\big[\lambda\delta[f] + (1-\lambda)\delta[g]\big] + (1-\kappa)P.$$

Corollary

$\succsim$ satisfies A1–A7 + A9 $\iff$ $\succsim$ has an OAA representation.

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ABSOLUTE AMBIGUITY AVERSION

Definition

$\succsim$ is ***absolutely ambiguity-averse*** if

$\succsim$ is more ambiguity-averse than an SEU preference.

Proposition

A CAP $\succsim$ w/ $\langle u, (\mathbb{M}, c)\rangle$ is absolutely ambiguity-averse $\iff \bigcap \mathbb{M} \neq \emptyset$.

- $\bigcap \mathbb{M}$ is independent of the choice of rep. (even w/o convexity)

COMPARATIVE AVERSION OF CAP AGAINST MEU

Proposition

Let $\langle (u, (\mathbb{M}, c)) \rangle$ be a CAP rep. of $\succsim$, and (v, L) an MEU rep. of $\geqslant$.

The following are equivalent.

a. $u \approx v$ & $\bigcap \mathbb{M} \supseteq L$.

b. $\succsim$ *is more ambiguity-averse than* $\geqslant$.

c. $\succsim$ *is more averse to additional ambiguity than* $\geqslant$.

- Two comparatives coincide when the benchmark is MEU
  - They are different else

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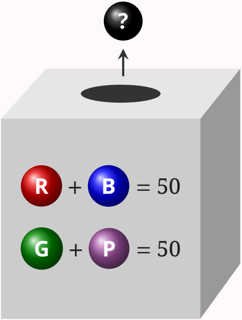
REFLECTION EXAMPLE

©MACHINA (2009)

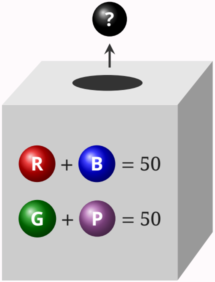


	R	B	G	P
f_1	100	200	100	0
f_2	100	100	200	0
f_3	0	200	100	100
f_4	0	100	200	100

- SEU + symmetry $\implies f_1 \sim f_2 \sim f_3 \sim f_4$
- Typical pattern: $f_1 \prec f_2$ & $f_3 \succ f_4$ $\langle\star\rangle$ ©L'Haridon · Placido (2010)
- $\langle\star\rangle$ *is inconsistent w/ uncertainty aversion* 😢

©Baillon et al. (2011)
*under natural assumptions

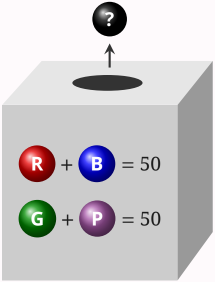
Reflection example vs. *uncertainty aversion*



	R	B	G	P
f_1	100	200	100	0
f_2	100	100	200	0
f_3	0	200	100	100
f_4	0	100	200	100

	R	B	G	P		R	B	G	P
f_1	0	100	100	0	+	100	100	0	0
f_2	0	0	200	0	+	100	100	0	0
f_3	0	200	0	0	+	0	0	100	100
f_4	0	100	100	0	+	0	0	100	100

REFLECTION EXAMPLE VS. *UNCERTAINTY AVERSION*



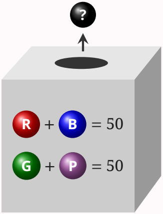
	R	B	G	P
f_1	100	200	100	0
f_2	100	100	200	0
$\hat{f}_3$	100	300	0	0
$\hat{f}_4$	100	200	100	0

- By *uncertainty aversion*,

$$f_2 \sim f_3 \sim \hat{f}_3 \implies \hat{f}_4 = f_1 = \frac{1}{2} f_2 + \frac{1}{2} \hat{f}_3 \succsim f_2 \sim \hat{f}_3$$

$\rightsquigarrow$ the opposite of $\langle\star\rangle$ — $f_1 \prec f_2$ & $f_3 \succ f_4$

Reflection example vs. CAP



	R	B	G	P
f_1	100	200	100	0
f_2	100	100	200	0
f_3	0	200	100	100
f_4	0	100	200	100

$\langle\star\rangle$ $f_1 \prec f_2$ $\quad f_3 \succ f_4$

- Identify each prior w/ $(\mu_B, \mu_G) \in [0, \frac{1}{2}]^2$ $\quad \rightsquigarrow (\mu_R, \mu_P) = (\frac{1}{2} - \mu_B, \frac{1}{2} - \mu_G)$
- $\mathbb{M} = \{\, M(\beta, \gamma) \subseteq \Delta(\Omega) \mid (\beta, \gamma) \in [0, 1]^2 \,\}$
- $c\colon [0,1]^2 \to \mathbb{R}_+$: LSC, $c(1,1) = 0$, strictly decreasing, symmetric

$$U_{\text{CAP}}(f_1) \leq U_{\text{CAP}}(f_2) = U_{\text{CAP}}(f_3) \geq U_{\text{CAP}}(f_4)$$

strict inequality for "not too large" c

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D 50–51 example

50–51 Example

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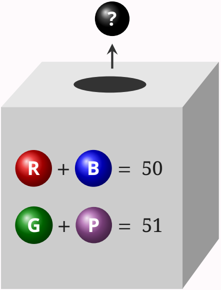


	R	B	G	P
f_5	202	202	101	101
f_6	202	101	202	101
f_7	303	202	101	0
f_8	303	101	202	0

- Machina's conjecture: $f_5 \succ f_6$ & $f_7 \prec f_8$ $\langle\spadesuit\rangle$
  - f_6 & f_8 have slight "objective advantage"
  - f_5 is not ambiguous, while f_7 is ambiguous
  - If you said $f_6 \succ f_5$, imagine 100 : 101 instead of 50 : 51…🤔

50–51 EXAMPLE VS. DSEU

- Identify each prior μ w/ $(\mu_{\mathsf{B}}, \mu_{\mathsf{G}})$ $\quad\rightsquigarrow\quad$ $(\mu_{\mathsf{R}}, \mu_{\mathsf{P}}) = (\frac{50}{101} - \mu_{\mathsf{B}}, \frac{51}{101} - \mu_{\mathsf{G}})$
- For $\bigcup \mathbb{M} \subseteq [0, \frac{50}{101}] \times [0, \frac{51}{101}]$,

$$U_{\mathsf{DSEU}}(f_5) = 151$$

$$U_{\mathsf{DSEU}}(f_6) = 151 + 101 \max_{M \in \mathbb{M}} \min_{\mu \in M} (-\mu_{\mathsf{B}} + \mu_{\mathsf{G}})$$

$$U_{\mathsf{DSEU}}(f_7) = 150 + 101 \max_{M \in \mathbb{M}} \min_{\mu \in M} (-\mu_{\mathsf{B}} + \mu_{\mathsf{G}})$$

$$U_{\mathsf{DSEU}}(f_8) = 150 + 202 \max_{M \in \mathbb{M}} \min_{\mu \in M} (-\mu_{\mathsf{B}} + \mu_{\mathsf{G}})$$

$$U_{\mathsf{DSEU}}(f_5) - U_{\mathsf{DSEU}}(f_6) = U_{\mathsf{DSEU}}(f_7) - U_{\mathsf{DSEU}}(f_8)$$

50–51 EXAMPLE VS. CAP

- $\mathbb{M} = \{ M(\beta,\gamma) \mid (\beta,\gamma) \in [0,1]^2 \}$

$$M(\beta,\gamma) = \left[(1-\beta)\{\tfrac{25}{101}\} + \beta[0, \tfrac{50}{101}]\right] \times \left[(1-\gamma)\{\tfrac{25.5}{101}\} + \gamma[0, \tfrac{51}{101}]\right]$$

- $c\colon [0,1]^2 \longrightarrow \mathbb{R}_+$: LSC, $c(1,1) = 0$, & strictly decreasing
- For each $\theta \in \mathbb{R}_{++}$, the preference w/ $(\mathbb{M}, \theta c)$ satisfies

$$U_{\text{CAP}}(f_5) = 151 \qquad U_{\text{CAP}}(f_6) = 151.5 - \phi_\theta(\tfrac{1}{2})$$

$$U_{\text{CAP}}(f_7) = 150.5 - \phi_\theta(\tfrac{1}{2}) \qquad U_{\text{CAP}}(f_8) = 151 - \phi_\theta(1)$$

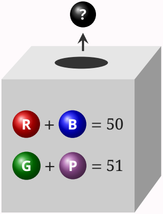
$$\phi_\theta(t) = \min_{(\beta,\gamma)\in[0,1]^2} [(50\beta + 51\gamma)t + \theta c(\beta,\gamma)]$$

$(\theta, t) \mapsto \phi_\theta(t)$ is concave

50–51 EXAMPLE VS. CAP

- $U_{\text{CAP}}(f_5) - U_{\text{CAP}}(f_6) = -0.5 + \phi_\theta(\tfrac{1}{2})$ — by concavity of ϕ_θ
- $U_{\text{CAP}}(f_7) - U_{\text{CAP}}(f_8) = -0.5 + (\phi_\theta(1) - \phi_\theta(\tfrac{1}{2})) \leq U_{\text{CAP}}(f_5) - U_{\text{CAP}}(f_6)$
- $\exists(\underline{\theta}, \overline{\theta}) \in \mathbb{R}^2_{++}$ s.t.

$$\underline{\theta} < \overline{\theta}, \qquad \theta > \underline{\theta} \iff f_5 \succ f_6, \qquad \theta < \overline{\theta} \iff f_7 \prec f_8$$



	R	B	G	P
f_5	202	202	101	101
f_6	202	101	202	101
f_7	303	202	101	0
f_8	303	101	202	0

$\langle\spadesuit\rangle$ $\quad f_5 \succ f_6$, $\quad f_7 \prec f_8$