Randomization and ambiguity perception

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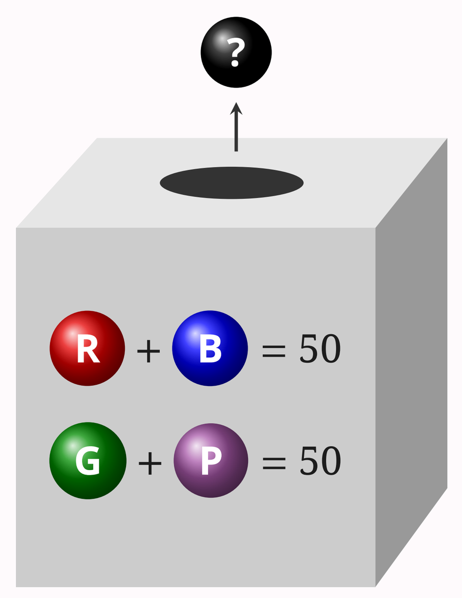
Summary

- We axiomatically study an ambiguity-averse DM who dislikes increasing in ambiguous events
- DM dislikes ***ex ante*** randomization—**A5**
- DM behaves as if optimizing her ambiguity perception at a cost
- Sharp identification & comparatives are obtained

Background

- People dislike unknown probabilities—***ambiguity aversion*** ©Ellsberg (1961)
  - Contradicts EU
  - Many models have been developed to explain AA (e.g., MEU model ©Gilboa · Schmeidler (1989))
- Machina's (2009) "reflection example"



	R	B	G	P
f_1	100	200	100	0
f_2	100	100	200	0
f_3	0	200	100	100
f_4	0	100	200	100

- Typical pattern: $f_1 \prec f_2$ & $f_3 \succ f_4$ $\langle\star\rangle$ ©L'Haridon · Placido (2010)

People dislike ***increases*** in relevant ambiguous events

- $\langle\star\rangle$ contradicts *uncertainty aversion*: 😢 ©Baillon · L'Haridon · Placido (2011) *under natural assumptions

$$f \sim g \implies \lambda f + (1-\lambda) g \succsim f \quad \forall \lambda \in [0,1]$$

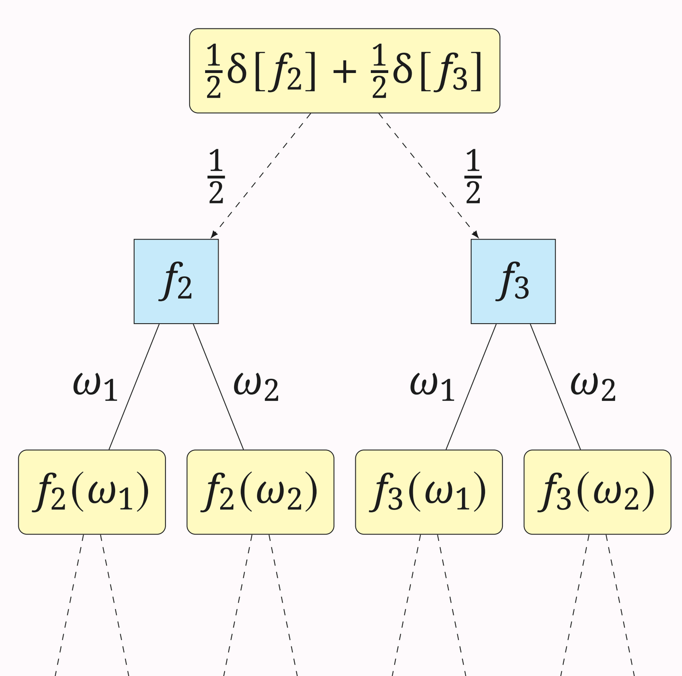
Randomization & ambiguous events

- Mixing bets has two conflicting effects e.g., $\frac{1}{2}f_2 + \frac{1}{2}f_3 = (50, 150, 150, 50)$

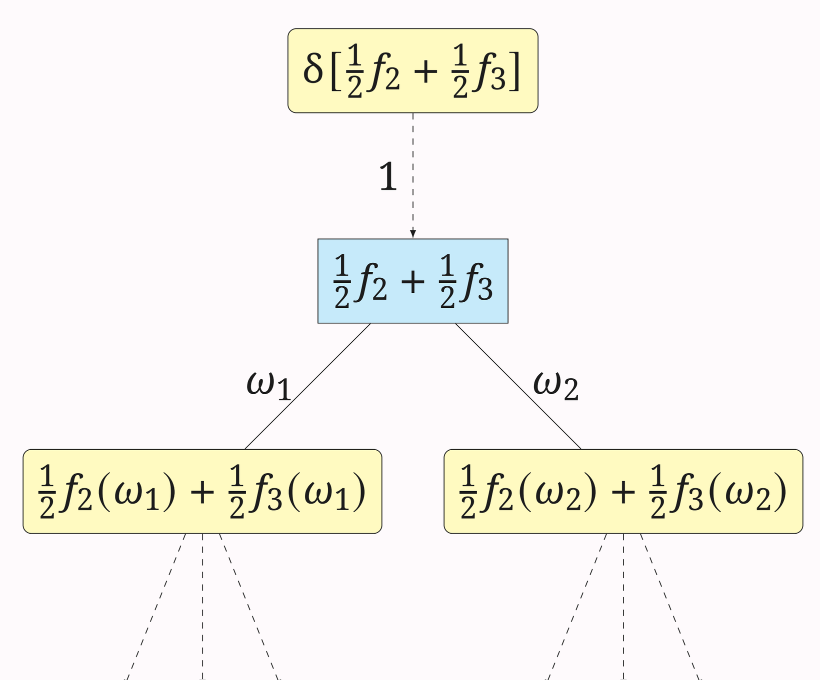
  1. Payoff smoothing 😌
  2. Increasing relevant ambiguous events 😔

- In a lottery ***over*** bets, payoffs are not smoothed + both **RB** & **GP** are relevant

$$P = \frac{1}{2}\delta[f_2] + \frac{1}{2}\delta[f_3] \neq \frac{1}{2}f_2 + \frac{1}{2}f_3 \implies f_2 \sim f_3 \succsim P \quad \& \quad \frac{1}{2}f_2 + \frac{1}{2}f_3 \succsim P$$



Ex ante randomization



Ex post randomization

Aversion to many ambiguous events $\rightsquigarrow$ *ex ante aversion to randomization*

Characterization

A1, A2, A7—Basic axioms

Weak order + nondegeneracy + continuity + monotonicity + unboundedness

◆ *Attitudes toward randomization timing*

A3—Attraction to ex post randomization

$\kappa\delta[\lambda f + (1-\lambda)g] + (1-\kappa)P \succsim \kappa[\lambda\delta[f] + (1-\lambda)\delta[g]] + (1-\kappa)P.$

A4—Indifference to randomization timing of constant acts

$\kappa\delta[\lambda f + (1-\lambda)p] + (1-\kappa)P \sim \kappa[\lambda\delta[f] + (1-\lambda)\delta[p]] + (1-\kappa)P.$

◆ *Attitudes toward ex ante randomization*

A5—Ex ante aversion to randomization

$P \succsim Q \implies P \succsim \lambda P + (1-\lambda)Q.$

A6—Independence of constant acts

$\lambda P + (1-\lambda)\delta[p] \succsim \lambda Q + (1-\lambda)\delta[p] \implies \lambda P + (1-\lambda)\delta[q] \succsim \lambda Q + (1-\lambda)\delta[q].$

Representation theorem

$\succsim$ satisfies A1–A7 $\iff$ $\succsim$ has a CAP representation.

- Replacing A5 + A6 w/ *independence* $\rightsquigarrow$ MEU

Identification

- A CAP representation is ***convex*** if $\mathbb{M}$ and c are convex

Identification theorem

Every CAP preference has convex representations $\langle u, (\mathbb{M}^*, c^*)\rangle$ & $\langle u, (\mathbb{M}^{**}, c^{**})\rangle$ s.t. $\mathbb{M}^* \subseteq \mathbb{M} \subseteq \mathbb{M}^{**}$ and $c = c^{**}|_{\mathbb{M}}$ for each convex representation $\langle u, (\mathbb{M}, c)\rangle$.

- Every CAP preference the smallest and largest convex representations
- $(u, \mathbb{M}^*)$ represents the ***expected utility core of*** $\succsim$:

$$\lambda P + (1-\lambda)R \succsim \lambda Q + (1-\lambda)R \qquad \forall (\lambda, R) \in (0,1] \times \Delta_s(\mathcal{F})$$
$$\iff \int \Big(\min_{\mu \in M} \int u \circ f \, d\mu\Big) dP(f) \geq \int \Big(\min_{\mu \in M} \int u \circ f \, d\mu\Big) dQ(f) \quad \forall M \in \mathbb{M}^*$$

- Our comparative shows the behavioral meaning:

more ambiguity-averse $\iff$ bigger $\mathbb{M}^{**}$ + higher c^{**}

absolute ambiguity-averse $\iff$ $\bigcap \mathbb{M} \neq \emptyset$

Ex ante randomization is important for identification

Let $\Omega = \{0,1\}$ and $X = \mathbb{R}$. Identify $\mu \in [0,1]$ w/ the prob. of $\omega = 1$.

Consider $\succsim_1$ and $\succsim_2$ represented by $\langle u, (\mathbb{M}_i, c_i)\rangle$:

$$u(x) = x, \qquad \mathbb{M}_1 = \{\{\tfrac{1}{2}\}\}, \qquad \mathbb{M}_2 = \{[0, \tfrac{1}{2}], [\tfrac{1}{2}, 1]\}, \qquad c_1 = c_2 \equiv 0.$$

While $\succsim_2$ is averse to ex ante randomization, $\succsim_1$ is neutral.

$\succsim_1$ and $\succsim_2$ coincide on the set of acts. ◊

Model

- Ω: finite set of ***states***
- $\Delta_s(X) = \{\ldots, p, q, \ldots\}$: set of *simple* distr.s of ***outcomes***
- $\mathcal{F} = \Delta_s(X)^\Omega = \{\ldots, f, g, \ldots\}$: set of ***acts***
- $\succsim$: DM's preference over lotteries of acts $\Delta_s(\mathcal{F}) = \{\ldots, P, Q, \ldots\}$

Definition

A ***costly ambiguity perception representation of*** $\succsim$ is $\langle u, (\mathbb{M}, c)\rangle$ s.t.

- $u\colon \Delta_s(X) \to \mathbb{R}$ is surjective & mixture linear;
- $\mathbb{M}$ is a compact set of nonempty closed convex subsets of $\Delta(\Omega)$;
- $c\colon \mathbb{M} \to \mathbb{R}_+$ is a lower semicontinuous function w/

  1. $\min c(\mathbb{M}) = 0$; 2. $M \subseteq M' \implies c(M) \geq c(M')$;

- $\succsim$ is represented by

$$U_{\mathrm{CAP}}(P) = \max_{M \in \mathbb{M}} \Big[\int \Big(\min_{\mu \in M} \int u \circ f \, d\mu\Big) dP(f) - c(M) \Big].$$

- $\mathbb{M} = \{M\} \iff U(P) = \max_{M \in \mathbb{M}} \Big[\int \Big(\min_{\mu \in M} \int u \circ f \, d\mu\Big) dP(f) - c(M) \Big]$

CAP model explains $\langle\star\rangle$

Identify each prior w/ $(\mu_{\mathbf{B}}, \mu_{\mathbf{G}}) \in [0, \frac{1}{2}]^2$. $\rightsquigarrow (\mu_{\mathbf{R}}, \mu_{\mathbf{P}}) = (\frac{1}{2} - \mu_{\mathbf{B}}, \frac{1}{2} - \mu_{\mathbf{G}})$

Let $\mathbb{M} = \{\, M(\beta, \gamma) \subseteq \Delta(\Omega) \mid (\beta, \gamma) \in [0,1]^2 \,\}$, where

$$M(\beta, \gamma) = \underbrace{\Big[(1-\beta)\{\tfrac{1}{4}\} + \beta[0, \tfrac{1}{2}]\Big]}_{\textbf{RB}\text{ dimension}} \times \underbrace{\Big[(1-\gamma)\{\tfrac{1}{4}\} + \gamma[0, \tfrac{1}{2}]\Big]}_{\textbf{GP}\text{ dimension}}.$$

If $c\colon [0,1] \to \mathbb{R}_+$ is symmetric, strictly decreasing, LSC, and $c(1,1) = 0$, then

$$U_{\mathrm{CAP}}(f_1) = U_{\mathrm{CAP}}(f_4) = 100 - \min_{(\beta,\gamma) \in [0,1]^2} [25(\beta + \gamma) + c(\beta, \gamma)],$$
$$U_{\mathrm{CAP}}(f_2) = U_{\mathrm{CAP}}(f_3) = 100 - \min_{\gamma \in [0,1]} [50\gamma + c(\mathbf{1}, \gamma)].$$

Thus, $U_{\mathrm{CAP}}(f_1) \leq U_{\mathrm{CAP}}(f_2) = U_{\mathrm{CAP}}(f_3) \geq U_{\mathrm{CAP}}(f_4)$. (strict inequality for not too large c) ◊